Dimensional Analysis

Throughout your study of science, it is important that a unit accompanies all measurements. Keeping track of the units in problems can help you convert one measured quantity into its equivalent quantity of a different unit, or help set up a calculation without the need for a formula.

In conversion problems, equality statements such as "1 foot = 12 inches" are made into fractions and then strung together in such a way that all units except the one desired are canceled out of the expression. Remember that defined numbers, such as 1 foot or 12 inches, are exact numbers and thus will not affect the number of significant digits in your answer. This method is also known as the "Factor-Label" method or the "Unit-Label" method.

To set up a conversion problem, follow these steps:

1. Think about and write down all the "=" statements you know that will help you get from your current unit to the new unit.
2. Make fractions out of your "=" statements. There should be two fractions for each "=" and they will be reciprocals of each other.
3. Begin solving the problem by writing the given amount with units on the left and then choose the fractions that will let a numerator unit be canceled with a denominator unit, and vice versa.
4. Using your calculator, read from left to right and enter the numerator and denominator numbers in order. Precede each numerator number with a multiplication sign and each denominator number with a division sign. Alternatively, you could enter all of the numerators, each separated by a multiplication sign, and then all of the denominators, each separated by a division sign.
5. Round your calculator's answer to the correct number of significant digits based on the number with the least number of significant digits in your original problem.

Example 1

How many inches are in 1.25 miles?

$$1\,\text{ft} = 12\,\text{in.} \Rightarrow \frac{1\,\text{ft}}{12\,\text{in.}} \text{ or } \frac{12\,\text{in.}}{1\,\text{ft}}$$

$$5280\,\text{ft} = 1\,\text{mile} \Rightarrow \frac{5280\,\text{ft}}{1\,\text{mile}} \text{or} \frac{1\,\text{mile}}{5280\,\text{ft}}$$

$$1.25\,\text{mile} \times \frac{5280\,\text{ft}}{1\,\text{mile}} \times \frac{12\,\text{in.}}{1\,\text{ft}} = 79{,}200\ \text{in.}$$

As problems get more complex, the measurements may contain fractional units or exponential units. To handle these situations, treat each unit independently. Structure your conversion factors to ensure that all the given units cancel out with a numerator or denominator as appropriate and that your answer ends with the appropriate unit. Sometimes information given in the problem is an equality that will be used as a conversion factor.

Squared and cubed units are potentially tricky. Remember that a square centimeter (cm^2) is really cm × cm. If we need to convert square centimeters to square millimeters (mm^2), we need to use the conversion factor of 1 cm = 10 mm twice so that both centimeter units cancel out.

Example 2

Suppose your automobile tank holds 23 gallons and the price of gasoline is 33.5¢ per liter. How many dollars will it cost you to fill your tank?

From a reference table, we find 1 L = 1.06 qt and 4 qt = 1 gal. We should recognize from the problem that the price is also an equality (33.5¢ = 1 L) and we should know that 100¢ = $1.

Setting up the factors, we find

$$23\,\text{gal.} \times \frac{4\,\text{qt}}{1\,\text{gal.}} \times \frac{1\,\text{L}}{1.06\,\text{qt}} \times \frac{33.5¢}{1\,\text{L}} \times \frac{\$1}{100¢} = \$29$$

In your calculator, enter

$$23 \times 4 \div 1.06 \times 33.5 \div 100 = 29.0754717$$

However, because the given value of 23 gallons has only two significant digits, your answer should be rounded to $29.

Example 3

One liter is exactly 1000 cm^3. How many cubic inches are there in 1.0 liters?

We should know that 1000 cm^3 = 1 L, and from a reference table we find that 1 in. = 2.54 cm. Setting up the factors, we find

$$1.0\,\text{L} \times \frac{1000\,(\text{cm} \times \text{cm} \times \text{cm})}{1\,\text{L}} \times \frac{1\,\text{in.}}{2.54\,\text{cm}} \times \frac{1\,\text{in.}}{2.54\,\text{cm}} \times \frac{1\,\text{in.}}{2.54\,\text{cm}} = 61\ \text{in.}^3$$

(The answer must have two significant digits because our given value 1.0 L contains two significant digits.)

As you become more comfortable with the concept of unit cancellation, you will find that it is a very handy tool for solving problems. By knowing the units of your given measurements and by focusing on the units of the desired answer, you can derive a formula and correctly calculate an answer. This is especially useful when you have forgotten (or never knew) the formula.

Even though you may not know the exact formula for solving this problem, you should be able to match the units up in such a way that only your desired unit does not cancel out.

Example 4

What is the volume in liters of 1.5 moles of gas at 293 K and 1.10 atm of pressure? The ideal gas constant is $\frac{0.0821\,\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$.

It is not necessary to know the formula for the ideal gas law to solve this problem correctly. Working from the constant (because it sets the units), we must cancel out every unit except liters. Doing this shows us that moles and Kelvin must be in the numerator and atmospheres in the denominator:

$$V = \frac{(1.5\text{ mol})\left(0.0821\frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)(293\text{ K})}{1.10\text{ atm}} = 32.8\text{ L, or }33\text{ L}$$

The answer is reported to two significant digits because our least accurate measurement (1.5 mol) has only two significant digits.

Note: Never rely on the number of significant digits in a constant to determine the number of significant digits for reporting your answer. Consider only the number of significant digits in given or measured quantities.